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SHORTER COMMUNICATIONS

LAMINAR MIXED CONVECTION FROM A HORIZONTAL ROTATING DISC

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NOMENCLATURE

F, G, H,	dimensionless axial velocity, tangential velocity and temperature;
g,	acceleration due to gravity;
r, θ, z,	radial, tangential and axial co-ordinate;
T,	temperature;
u, v, w, Ω,	radial, tangential, axial and angular velocity;
α,	coefficient of thermal expansion;
κ,	thermal diffusivity;
ν,	kinematic viscosity
ζ,	dimensionless axial co-ordinate;
Gr,	Grashof number $= g\alpha(T_w^* - T_0)r^3/\nu^2$;
Pr,	Prandtl number $= \nu/\kappa$;
Re,	Reynolds number $= r^2\Omega/\nu$.

Subscripts

$0, w, \infty$,	value at the origin, $z = 0$ and $z \to \infty$, respectively;
$1, 2$,	radial, axial component.

1. INTRODUCTION

IN THE presence of buoyancy, the von Kármán [1] similarity solution to the problem of laminar flow and heat transfer from a horizontal rotating disc requires that the radial temperature distribution be quadratic (Duncan [2], Luk, Millsaps and Pohlhausen [3], Rotem and Claassen [4]). Luk *et al.* [3] have discussed the buoyant and dissipative flow above an isothermal rotating disc whereas Rotem and Claassen [4] have investigated the effect of slow rotation on the natural convection above a heated horizontal surface. Duncan [2] has considered the low Rossby number regime of buoyancy-induced convective perturbations of a rigid-body rotation of a stably stratified fluid in the gap between a top disc heated quadratically and an insulated lower disc.

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In this paper, the effect of buoyancy on the flow and heat transfer above a heated, horizontal, rotating disc is investigated. The flow is assumed steady, laminar, incompressible and non-dissipative. The transport properties of the fluid are considered constant. The Boussinesq approximation is applied to the governing equations, which are then solved by a numerical method for a value 0·701 of Prandtl number and $Gr/Re^{\frac{3}{2}}$ between 0 and 1. Buoyancy is seen to alter only the hydrostatic pressure distribution over an isothermal disc whereas, with quadratic surface temperature, the influence of buoyancy is more on the secondary motion over the disc than on the rotation of, or the heat transfer by, the fluid. Buoyancy and rotation aid each other above heated, and below cooled, discs while they are in opposition below heated, and above cooled, discs.

2. THE GOVERNING EQUATIONS

In an inertial cylindrical frame of reference, with its z-axis oriented antiparallel to gravity, the Boussinesq equations under the similarity assumptions of von Kármán [1] can be reduced to (Duncan [2]):

$$-\frac{w}{2}\frac{d^3w}{dz^3} - 2\frac{v}{r}\frac{d}{dz}\left(\frac{v}{r}\right) = -g\alpha T_1(z) - \frac{\nu}{2}\frac{d^4w}{dz^4} \quad (2.1)$$

$$-\frac{dw}{dz}\left(\frac{v}{r}\right) + w\frac{d}{dz}\left(\frac{v}{r}\right) = \nu\frac{d^2}{dz^2}\left(\frac{v}{r}\right) \quad (2.2)$$

$$-\frac{dw}{dz}T_1 + w\frac{dT_1}{dz} = \kappa\frac{d^2T_1}{dz^2} \quad (2.3)$$

$$w\frac{dT_2}{dz} = \kappa\left(\frac{d^2T_2}{dz^2} + 2T_1\right). \quad (2.4)$$

The temperature distribution:

$$(T - T_\infty) = \tfrac{1}{2}r^2T_1(z) + T_2(z). \quad (2.5)$$

The boundary conditions are:

$$w(0) = \frac{dw}{dz}(0) = 0, v(0) = r\Omega; (T(0) - T_\infty)$$

$$= \tfrac{1}{2}r^2 T_1(0) + T_2(0)$$

$$w(\infty) = \text{constant}, v(\infty) = T_1(\infty) = T_2(\infty) = 0. \quad (2.6)$$

These equations are now non-dimensionalized using $\nu^{\frac{1}{2}}\Omega^{-\frac{1}{2}}$, $\nu^{\frac{1}{2}}\Omega^{\frac{1}{2}}$, $r\Omega$, $T_w^* - T_0$ as the characteristic scales for length, axial velocity, tangential velocity, and temperature difference, respectively. The dimensionless equations are:

$$F'''' - FF''' = 4GG' - 2\frac{Gr}{Re^{\frac{3}{2}}}H_1 \quad (2.7)$$

$$G'' = G'F - GF' \quad (2.8)$$

$$H_1'' = Pr(H_1'F - H_1F') \quad (2.9)$$

$$H_2'' = PrH_2'F - 2H_1 \quad (2.10)$$

where the primes denote differentiation w.r.t. ζ and T_w^* is the wall temperature at $r = (2\nu/\Omega)^{\frac{1}{2}}$ in order to make $H_1(0) = 1$. The boundary conditions:

$$F(0) = 0 = F'(0), G(0) = 1 = H_1(0), H_2(0)$$
$$= (T_0 - T_\infty)/(T_w^* - T_0) \quad (2.11)$$

$$F(\infty) = \text{constant}, G(\infty) = H_1(\infty) = H_2(\infty) = 0. \quad (2.12)$$

There are three parameters $Gr/Re^{\frac{3}{2}}$, Pr, $(T_w^* - T_0)/(T_0 - T_\infty)$ in these equations that could be prescribed arbitrarily. The results here are restricted to (i) a value of $Pr = 0{\cdot}701$ corresponding to air, (ii) no axial temperature variation i.e. $T_0 = T_\infty$, and (iii) values of $Gr/Re^{\frac{3}{2}}$ between 0 and 1.

3. NUMERICAL SOLUTIONS

The equations (2.7)–(2.12) have been solved by Sreenivasan [6] using an initial-value technique or the shooting method. The integration is based on a predictor-corrector method due to Hamming (Ralston and Wilf [5]). By an integral check, analogous to those used by Luk *et al.* [3], five digits were found to be significant in the initial values $F''(0)$, $F'''(0)$, $G'(0)$, $H_1'(0)$ and $H_2'(0)$ reported in Table 1.

4. DISCUSSION

The similarity transformation of von Kármán applies to the combined convection problem only when the horizontal distribution of temperature in the fluid is quadratic. This fact can be deduced directly from the momentum equations using Batchelor's [7] approach. One consequence of this is that the effect of buoyancy in the flow over an isothermal disc is trivial in that it alters only the hydrostatic pressure distribution in the fluid. There is no change in the flow or heat transfer. Experimental verification of this result is likely to be difficult since the similarity conditions with a finite-size rotating disc are unsatisfactory. In addition, convective instability and edge effects could cause large

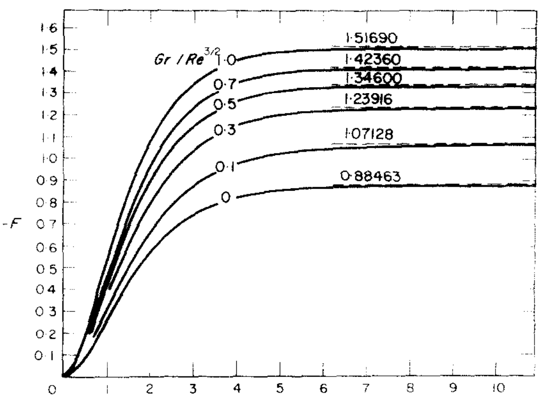

FIG. 1. Axial velocity profile.

Table 1. Initial values

Sl. No.	$\frac{Gr}{Re^{\frac{3}{2}}}$	$-F''(0)$	$F'''(0)$	$-G'(0)$	$-H_1'(0)$	$H_2'(0)$
1	0·0	1·020466	2·0	0·615922	0·515042	2·375666
2	0·1	1·172468	2·287934	0·661950	0·557397	2·088292
3	0·2	1·299832	2·539832	0·693886	0·585853	1·952097
4	0·3	1·414110	2·773064	0·719636	0·608565	1·860666
5	0·4	1·519425	2·993742	0·741512	0·627741	1·791978
6	0·5	1·618015	3·205063	0·760691	0·644484	1·737054
7	0·6	1·711273	3·408980	0·777861	0·659430	1·691618
8	0·7	1·800157	3·606898	0·793440	0·672953	1·652917
9	0·8	1·885328	3·799573	0·807778	0·685387	1·619129
10	0·9	1·967308	3·987807	0·821067	0·696893	1·589296
11	1·0	2·046498	4·172148	0·833469	0·707620	1·562622

discrepancies. The only experimental work known so far using a horizontal disc is due to Young [8] and it reports an increase in heat transfer. Cobb and Saunders [9] and Richardson and Saunders [10] consider Young's results to be subject to large experimental errors.

It can also be seen clearly that buoyancy gives rise to a radial pressure gradient, in the case of quadratic temperature distribution, which is favorable above a heated or below a cooled disc but adverse below a heated or above a cooled disc. The centrifugal forces in the boundary layer are assisted by buoyancy in the former situation while they are resisted in the latter. Thus in the first case, the secondary flow should increase. As the energy transfer, by virtue of axial symmetry, is due entirely to the secondary motion there should be an increase in heat transfer. In the other case, the adverse pressure gradient reduces and retards the secondary flow and also decreases the heat transfer. Thus when buoyancy is sufficiently large, the secondary motion could be largely suppressed and separation tendencies might develop.

The numerical results presented in Table 1 confirm these observations. In the flow aided by buoyancy, the axial velocity, Fig. 1, increases by 75 per cent and the frictional resistance to the secondary motion is doubled when $Gr/Re^{3/2} = 1$. The radial velocity profile, Fig. 2, gradually assumes a shape typical of buoyancy dominated flow as $Gr/Re^{3/2}$ increases. The frictional resistance to rotation and heat transfer due to T_1 increase only by about 35 per cent in the range (0, 1) of $Gr/Re^{3/2}$. The profiles of tangential velocity

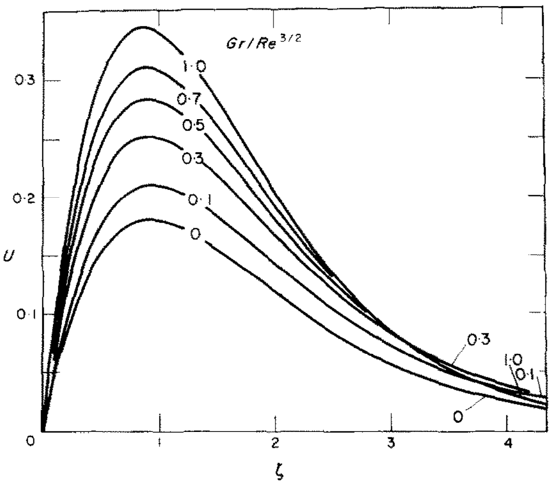

FIG. 2. Radial velocity profile.

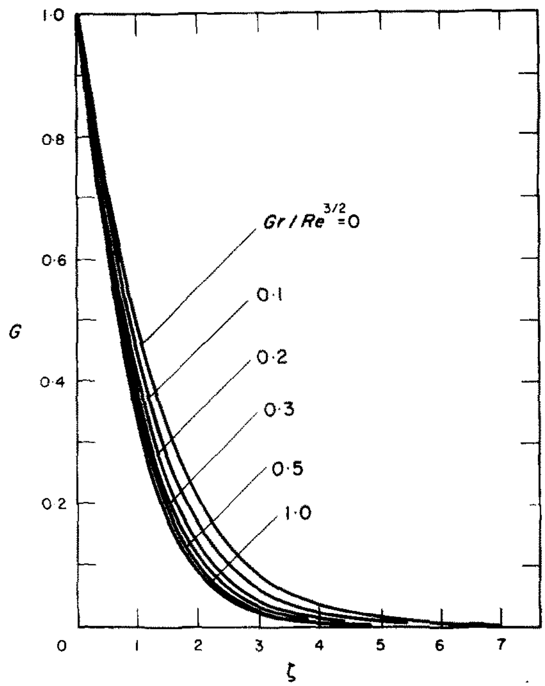

FIG. 3. Tangential velocity profile.

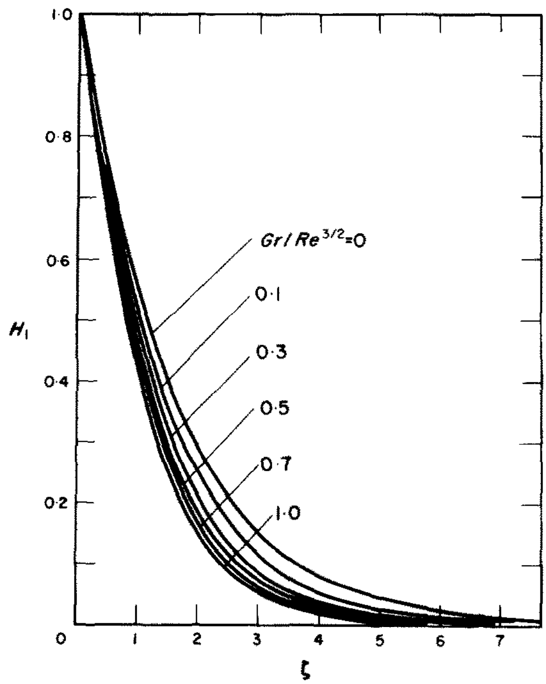

FIG. 4. Temperature profile H_1.

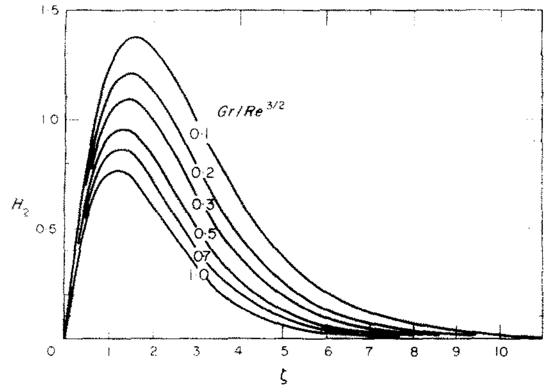

FIG. 5. Temperature profile H_2.

and horizontal temperature gradient, Figs. 3 and 4, do not change significantly. Thus the effect of buoyancy is observed to be more pronounced on the secondary motion than on the rotation of the fluid. Likewise, the change in frictional resistance to the radial flow is larger than the change in heat transfer. The heat transfer due to T_2 is into the disc in the case $T_0 = T_\infty$. But this is only a small fraction of the total heat transfer at large distances from the axis. The component T_2 is driven mainly by the radial conduction of heat due to T_1 when $T_0 = T_\infty$. This energy can only go into the wall. The profile of T_2, Fig. 5, gradually collapses as $Gr/Re^{\frac{3}{2}}$ increases.

At $Gr/Re^{\frac{3}{2}} = 1$, the velocities induced by buoyancy are of the same order as those due to rotation. Hence, to get results further, the equations should be recast into a form based on quantities characterizing natural convection flow on the lines followed by Rotem and Claassen [4].

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VIEW FUNCTION IN GENERALIZED CURVILINEAR COORDINATES FOR SPECULAR REFLECTION OF RADIATION FROM A CURVED SURFACE

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NOMENCLATURE

- dS_0, dS_1, dS_2, element of area of emitter, reflector, and receiver;
- $(x_0, y_0, z_0), (x, y, z), (X, Y, Z)$, coordinates of dS_0, dS_1 and dS_2, respectively;
- r, distance from emitter, dS_0, to reflector, dS_1;
- r', distance from reflector, dS_1, to receiver, dS_2;
- ρ, reflectivity of reflector;